

## Symmetrical Scales Worksheet

Key:

- *Rotation* or *R* means the form is a mode of some other configuration. We are searching for "parent" configurations, so these are eliminated.
- *Excluded 11* means there are two half-steps in a row in the configuration.
- *Splitting 3* means the minor third can be divided into a whole and half-step, creating an already accounted-for configuration. Many of the modes are really another scale missing one tone.

### 6 Intervals

Requirement: six numbers, each of which could be 1, 2 or 3 that add to 12.

1 1 1 1 1 1 =6  
1 1 1 1 1 2 =7  
1 1 1 1 1 3 =8  
1 1 1 1 2 2 =8  
1 1 1 1 2 3 =9  
1 1 1 1 3 3 =10  
1 1 1 2 2 2 =9  
1 1 1 2 2 3 =10  
1 1 1 2 3 3 =11  
**1 1 1 3 3 3 =12**

1 1 2 2 2 2 =10  
1 1 2 2 2 3 =11  
**1 1 2 2 3 3 =12**

1 1 3 3 3 3 =14

1 2 2 2 2 2 =11  
**1 2 2 2 2 3 =12**

1 2 3 3 3 3 X

1 3 3 3 3 3 X

**2 2 2 2 2 2 =12**

The result: There are **four groups**:

**Group: 1 1 1 3 3 3:** Augmented Scale. This can only be configured as 1 3 1 3 1 3, with two modes.

**Group: 2 2 2 2 2 2:** Whole tone scale, only one configuration, only one mode.

**Group: 1 2 2 2 2 3**

Configurations:

1 2 2 2 2 3: Rotates to: 222312. Lydian-Dominant missing 5. Related to Melodic Minor. Split the 3:

12222(12) rotates to 222(12)12 or Lydian-Dominant  
 1 2 2 2 3 2 Rotates to 222321 Lydian no 5th. Subset of Lydian, or Lydian Augmented.  
 1 2 2 3 2 2 Rotates to 221223 Subset of Major, no 7th.  
 1 2 3 2 2 2 Rotates to 221232 Mixolydian no 6th, or 232221 Major or Melodic Minor no 3rd.  
 1 3 2 2 2 2 Rotates to 322221 Melodic Minor, no 2nd. splitting 3: (21)22221 Melodic Minor.  
 Splitting 3 (12)22221 creates sequential half-steps.

**Group: 1 1 2 2 3 3**

Configurations:

1 1 n n n n excluded 11  
 1 2 1 2 3 3 (See below)  
 1 2 1 3 2 3 : Can be subset of 121(21)2(12) Diminished Scale. Dividing the 3 any other way results in sequential half-steps.  
 1 2 1 3 3 2 (See below)  
 1 2 2 1 3 3 (See below)  
 1 2 2 3 1 3 rotation: 2 2 3 1 3 1 = 1 2 3 5 b6 7 Harmonic Major missing 4. Adding #4 creates sequential half-steps.  
 1 2 3 1 2 3 Subset of 12(12)12(12) diminished scale.  
 1 2 3 1 3 2 Subset of 12(12)1(21)2 diminished scale  
 1 2 3 2 1 3 rotate to 2 3 2 1 3 1. 1 2 4 5 b6 7 Subset of Harmonic minor or Harmonic Major, if 3 is divided into 1,2 or 2,1.  
 1 2 3 2 3 1 excluded 11  
 1 2 3 3 1 2 R 121233  
 1 2 3 3 2 1 excluded 11  
 1 3 1 2 2 3 R 122313  
 1 3 1 2 3 2 R 123213  
 1 3 1 3 2 2 1 b2 3 4 b6 b7 Harmonic minor missing 5th.  
 1 3 2 1 2 3 R 123132  
 1 3 2 1 3 2 : Also can be considered subset of 1(21)21(21)2  
 1 3 2 2 1 3 R 131322  
 1 3 2 2 3 1 excluded 11  
 1 3 2 3 1 2 R 121323  
 1 3 2 3 2 1 excluded 11  
 1 3 3 1 2 2 R 122133  
 1 3 3 2 1 2 R 121332  
 1 3 3 2 2 1 excluded 11

\*Singling out the forms with sequential 33:

-3 3 1 1 2 2 excluded 11  
 -3 3 1 2 1 2 Subset of (12)(12)1212 diminished. Splitting 3 any other way creates sequential half-steps. 1122 1221 2112 2121 2211  
 -3 3 1 2 2 1 Splitting first 3: (21)31221 rotates to 221(21)31 Harmonic Major. Splitting second 3: 3(12)1221 rotates to 2122131 (this is odd: filled in tone is root; better as 1312122 Phrygian-Dom. Missing note is then the 4th.) Splitting both 3s: 1221, 2112, 1212, 2121, 2211, 1122 are out, create sequential half steps.  
 -3 3 2 1 1 2 excluded 11  
 -3 3 2 1 2 1 Subset of (21)(21)2121 diminished. Splitting 3s other ways creates sequential half-steps. 1122 1212 1221 2112 2211  
 -3 3 2 2 1 1 excluded 11

8 intervals

Requirement: 8 intervals adding to 12.

More the four ones will result in sequential half-steps.

**1111 2222** is only group of 1 and 2 that fits.

including 3: requires 7 intervals adding up to 9,

11 11 11 3 too many ones

11 11 12 2 still too many ones

11 11 n n n no combination of 2 and 3 for n will work, nor will any group with three or fewer ones.

Result: Only the 12121212 and 21212121 modes are possible with 8 notes. These are the two diminished scale forms.

So what's left?

If we omit exclusions, rotations and scales that are subsets of others, there are not really that many symmetrical scales:

1. Chromatic: 12 intervals of 1, only one mode.
2. Whole-tone: 6 intervals of 2, only one mode
3. Augmented: 6 intervals, alternating 1 and 3, two modes.
4. Diminished: 8 intervals, alternating 1 and 2, two modes.